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USING ABAQUS TO ANALYSE FATIGUE CRACK GROWTH UNDER THE COMBINED INFLUENCE OF RESIDUAL STRESS AND CYCLIC EXTERNAL LOAD

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ABSTRACT

Many analysts are keen to investigate cracks in components under static or cyclic external loading. In the latter case fatigue crack growth prediction is also of importance. It is known that surface treatment effects such as shot peening have a beneficial effect on component life due to the compressive residual stresses introduced in the vicinity of the surface. Such treatments are used in maintenance programmes to extend the life of components in service. The effect on crack growth rate can be dramatic with significant changes in the crack growth profiles and increase in fatigue life to failure. The approach discussed in this paper applies fracture mechanics techniques to establish crack growth rates based on detectable defect sizes above the crack initiation stage established by non-destructive inspection. Thereby, extended fatigue life can be predicted, extended inspection periods calculated and the retirement of some components may be avoided.

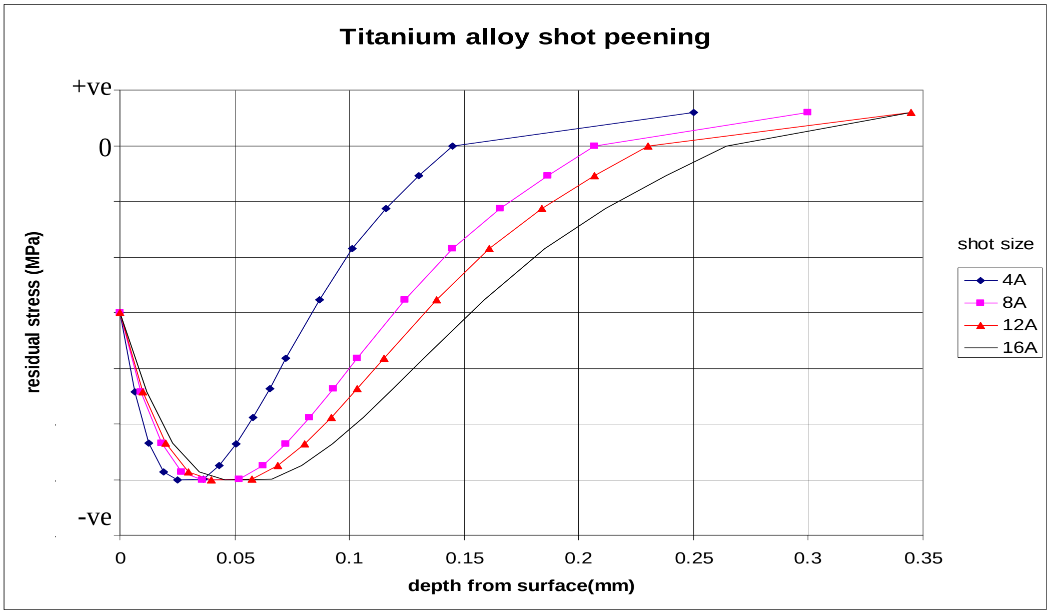
This paper demonstrates how the Abaqus DLOAD user subroutine can be used to include the effect of residual stresses from shot peening in the analysis of crack growth in 3D finite element models. The inclusion of these effects may be so beneficial that a crack will change from a “growing” state without residual stress to a “non-growing” i.e. below threshold state, if residual stress is included.

Initially the effect of residual stresses are presented using a linear fracture mechanics approach for a single edge notched specimen and a corner crack specimen. Subsequently, the non-linear effects of contact at the crack surfaces are modelled and the effects on the cyclic energy release rates are presented.

Examples are presented based on typical data for a titanium alloy.

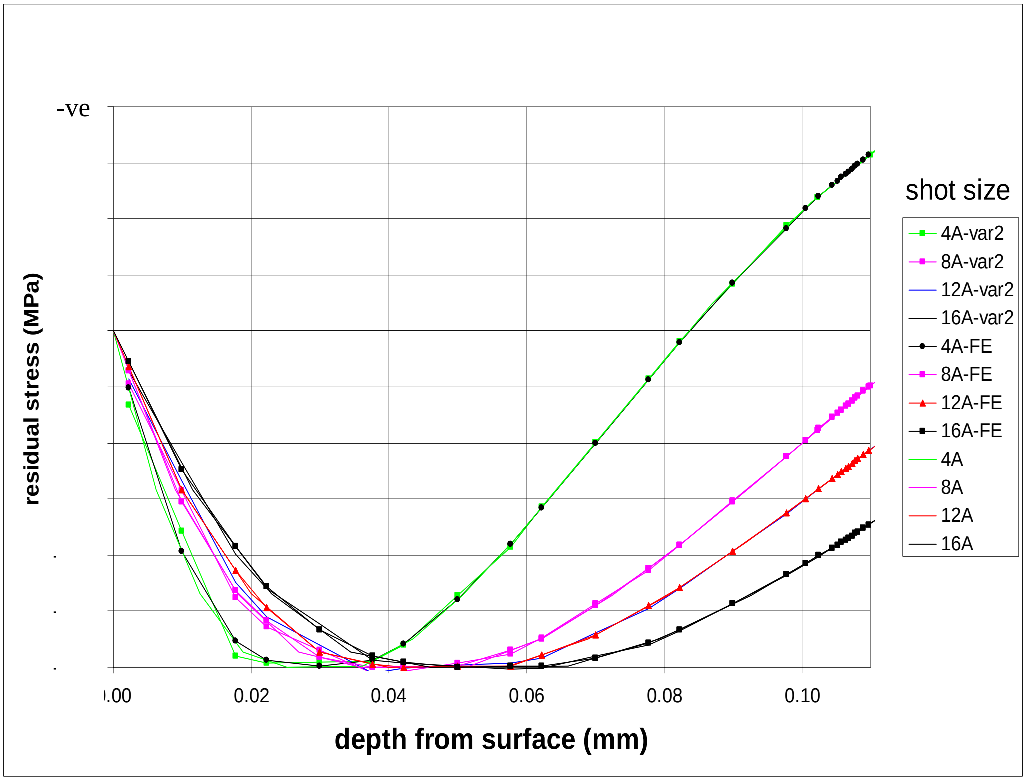
1 MODELLING THE RESIDUAL STRESS FIELD

A typical residual stress distribution normal to the surface resulting from surface shot peening is shown in Figure 1. It is noted that the stress distribution is self-equilibrating in both force and moment so that the stresses remote from the surface must eventually become compressive to balance the near-surface compressive stress field.



1. Residual stress distributions due to shot peening

The residual stress field is in reality applied all the way along the peened surface. However the principle of superposition can be used to apply the residual stresses directly to the surfaces of a contained crack using the Abaqus DLOAD user subroutine. The principle of superposition was verified using an Abaqus half symmetry 2-D finite element model of a single edged notched (SEN) specimen with a crack depth of 0.11 mm. The true uni-directional stress distribution was introduced as a subsurface nodal temperature distribution using *EXPANSION, TYPE=ORTHO and specifying a coefficient of expansion equal to the reciprocal of the Young's Modulus and setting Poisson Ratio to zero. The resulting pressure loads are shown in Figure 2 where each data point represents an integration point in the crack plane. In the legend, "var2" refers to thermal loading, "FE" refers to loading on the crack faces and the rest is the raw residual stress data. The average of the J-Integral second and third contour values for each loading method is shown in Table 1. Good agreement between the results verifies the principle of superposition.



2. Verification of the principle of superposition

Shot size	Energy release rates for directly applied residual stress (MPa√mm)	Energy release rates for residual stress applied to crack face (MPa√mm)	Percentage difference
	Average of Contours 2 & 3	Average of Contours 2 & 3	
4A	-2.1295	-2.1355	+0.281
8A	-3.638	-3.638	0.0
12A	-4.026	-4.018	-0.199
16A	-4.4085	-4.4045	-0.091

1. Comparison of averaged energy release rates to verify principle of superposition

1.1 Abaqus DLOAD user subroutine

The SEN crack is position along the x-axis with the x origin at the surface. The program uses a simple linear interpolation scheme between integration points. The subroutine uses a scale factor to examine the effects of stress relaxation on the initial residual stress field.

```
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C      SUBROUTINE DLOAD
C
C    Shot peen residual stress distributions for a Titanium alloy
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
      SUBROUTINE DLOAD(P,KSTEP,KINC,TIME,NOEL,NPT,LAYER,KSPT,COORDS,
     1                 JLTYP,SNAME)
C
      INCLUDE 'ABA_PARAM.INC'
C
      DIMENSION TIME(2),COORDS(3)
      CHARACTER*80 SNAME
C
c     shot peen data
      dimension sigma(19),x1(19),x2(19),x3(19),x4(19)
c     for all shot sizes
      data sigma/-306.0,-450.7,-545.3,-598.4,-612.0,-611.4,
     &           -586.3,-546.9,-498.5,-445.0,-389.6,-282.2,
     &           -188.9,-114.9,-55.1,0.0,61.2,0.0,-61.4/
c     for 4A shot
      data x1 /0.0,0.0063,0.0126,0.0189,0.0252,0.036225,
     &       0.04347,0.050715,0.05796,0.065205,0.07245,0.08694,0.10143,
     &       0.11592,0.13041,0.1449,0.25,1.811,100.0/
c     for 8A shot
      data x2/0.0,0.009,0.018,0.027,0.036,0.05175,0.0621,0.07245,
     &       0.0828,0.09315,0.1035,0.1242,0.1449,0.1656,0.1863,0.207,
```

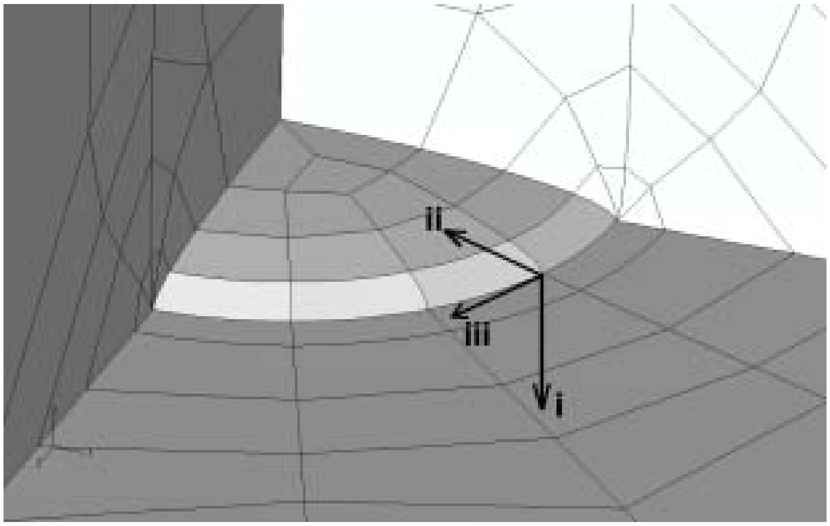
```
     &       0.3,2.597,100.0/
c     for 12A shot
      data x3/0.0,0.01,0.02,0.03,0.04,0.0575,0.069,0.0805,0.092,
     &       0.1035,0.115,0.138,0.161,0.184,0.207,0.23,0.345,2.89,
     &       100.0/
c     for 16A shot
      data x4/0.0,0.0115,0.023,0.0345,0.046,0.066125,0.07935,0.092575,
     &       0.1058,0.11902,0.13225,0.1587,0.18515,0.2116,0.23805,
     &       0.2645,0.345,3.3302,100.0/
      data scale/1.0/
C
      lunrep = 6
      lunmsg = 7
c
      select case (kstep)
      case(1)
            call zinterp(COORDS(1),x1,sigma,p)
      case(2)
            call zinterp(COORDS(1),x2,sigma,p)
      case(3)
            call zinterp(COORDS(1),x3,sigma,p)
      case(4)
            call zinterp(COORDS(1),x4,sigma,p)
      case default
          write(lunmsg,*)'unsupported ABAQUS step'
      end select
      p = scale*p
      write(lunmsg,10) kstep,coords(1),p,scale,noel,npt,jltyp
   10 format('kstep,x,p,scale,element,npt,jltyp=',i3,3e12.4,i8,2i3)
      RETURN
      END
C
C
C----------------------------------------------------------------------
C
C
      subroutine zinterp(c,x,sigma,p)
C
      INCLUDE 'ABA_PARAM.INC'
      dimension sigma(19),x(19)
c
      lunrep = 6
      lunmsg = 7
c
      if((c.lt.x(1)).or.(c.gt.x(19))) then
          write(lunrep,10) c
            write(lunmsg,10) c
   10     format('coordinate ',e12.4,
     &         ' is out of residual stress distribution range')
          stop
      endif
c
      do i= 2,19
         if(c.lt.x(i)) then
```

```
          p = sigma(i-1)+(c-x(i-1))*(sigma(i)-sigma(i-1))/(x(i)-x(i-1))
            return
        endif
     end do
     p = x(19)
     return
     end
```

1.2 Sign of the energy release rate

Abaqus calculates a positive value of the J-Integral regardless of the sign of the applied pressure distribution on the crack faces.

In order to resolve the sign of the J-Integral is is necessary to determine an “open” or “closed” status for each corner node on the crack front by extracting and processing nodal displacements. Each crack front node and the associated crack face quarter point nodes in the collapsed crack front elements are examined. A local coordinate system is created to allow calculation of relative opening displacements of pairs of nodes on either side of the crack face in local mode I, II and III orientations. An example is shown in Figure 3. The local mode I opening displacement, if positive, indicates an open crack at that point and a positive J-Integral. If the displacement is negative the crack is closed indicating a negative J-Integral.



3. Local opening directions at a crack front node

2 FATIGUE CRACK GROWTH

2.1 Crack growth data

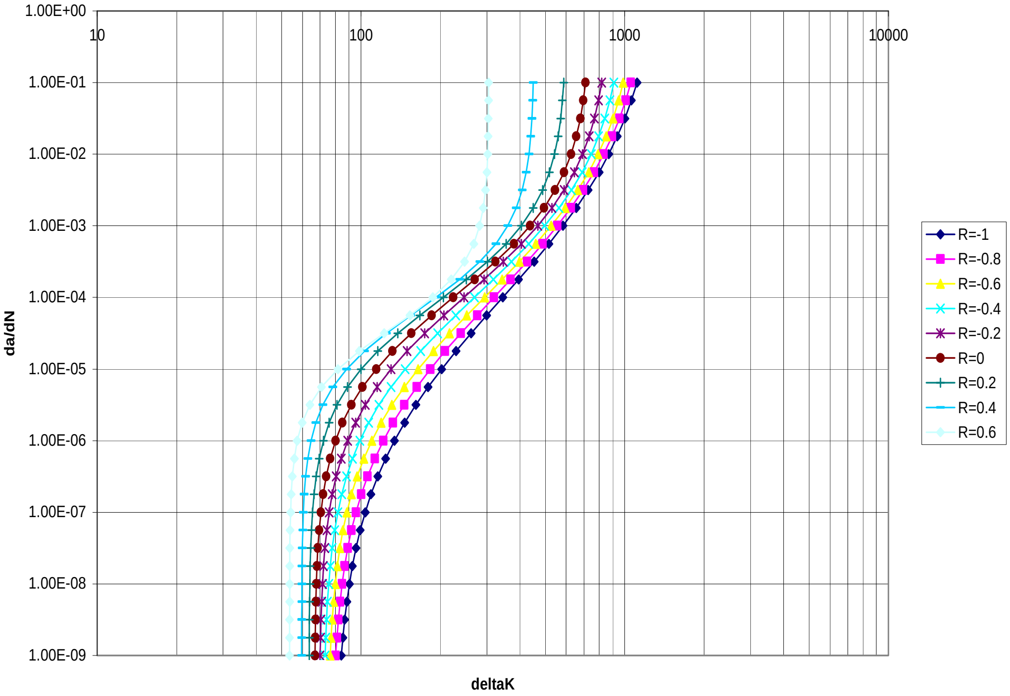
The crack growth data used in the 3-D FEA fatigue analyses was specified by the “TANH” equation commonly used for Titanium alloys which has the form:

$$\log\left(\frac{da}{dn}\right) = C_1 \langle \arctan h\left(C_2\left[\log(\Delta K) + C_3\right]\right)\rangle + C_4$$

in which:

$$C_i = A_{0i} + A_{1i} \times \log(1-R) + A_{2i} \times \left[\log(1-R)\right]^2 \qquad ,i = 1,2\ldots.4$$

da/dn is the crack growth rate, R is the stress ratio and “A” values are material constants. The crack growth curves are plotted in Figure 4.



4. Crack growth data for 3-D FEA fatigue analyses

2.2 Crack growth integration schemes

The fatigue loading comprises of the superposition of the static residual stresses and external cyclic loading. The two systems are combined to give and effective stress ratio, $R_{\text{effective}}$, that is a function of crack size, a.

$$R_{effective} = \frac{K_{extermnal\,\min} + K_{residual}}{K_{extermnal\,\max} + K_{residual}} \qquad (1)$$

The integration scheme uses a numerical forward predictor method in which it is assumed that dG/da is constant over each integration step (i to f) where G is the energy release rate:-

$$\frac{da}{dn} = f\left(\Delta\sqrt{G}\right) \qquad (2)$$

allowing a general integral to be written:

$$\int_i^f dn = \int_i^f \frac{1}{f\left(\Delta\sqrt{G}\right)} da \qquad (3)$$

where $$G_{\max f} = G_{\max i} + \left(\frac{dG_{\max}}{da}\right) da_{if} \qquad (4)$$

and $$\Delta\sqrt{G} = \left(G_{\max}^{1/2} - G_{\min}^{1/2}\right) = \Delta K\left(\frac{1-(\alpha\nu)^2}{E}\right)^{1/2}$$

ΔK = stress intensity factor range, E = Youngs Modulus, ν=Poisson Ratio,

α ranges from 0 for plane stress to 1 for plane strain conversion of crack growth data from ΔK to $\Delta\sqrt{G}$

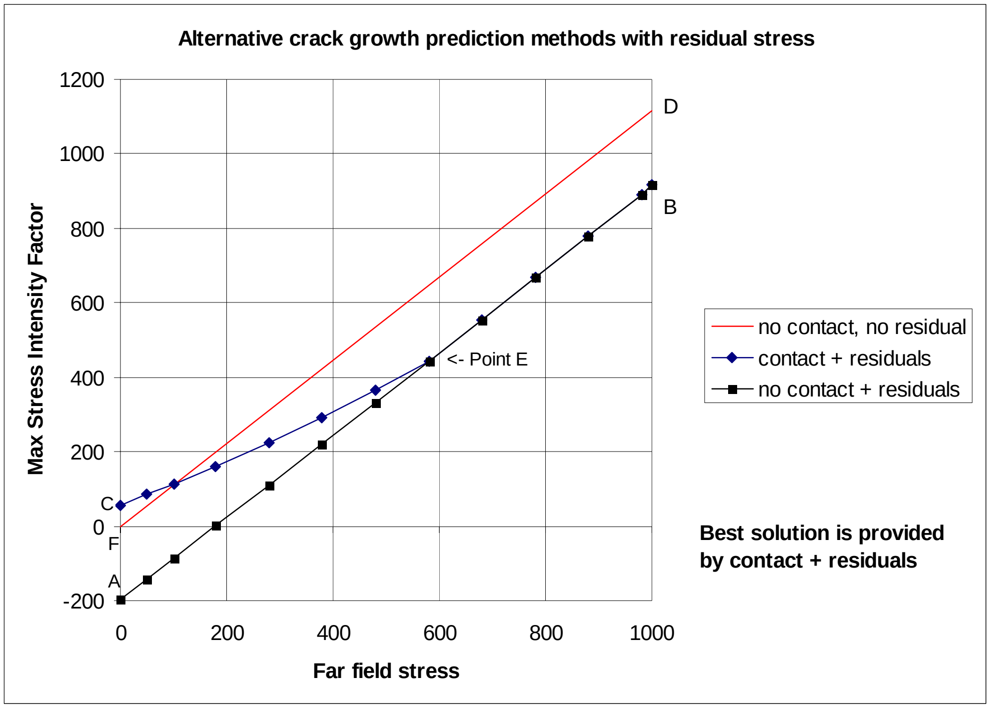
2.3 General response with residual stress and external cyclic loading

The typical stress intensity factor response at a crack front node in the presence of residual stresses is strongly influenced by contact (or partial contact) at the crack faces at lower external load levels and stress ratios as shown in Figure 5. In the figure:

- Point A represents the application of (compressive) residual stress in the absence of external loading.
- Line AB is the response to increasing external loading in the presence of the residual stress.
- Line FD is the response to external loading alone.

- Line CEB is a typical response to combined residual stress and external loading when contact at the crack faces is modelled in the FEA. This represents the “true” solution.

- Point E is located at the position when the external load is just sufficient to cause ALL the nodes on the crack faces to just become fully open.



5. Crack growth prediction methods in the presence of residual stress

Two methods were used to analyse the 3-D finite element models (FEA) in the presence of static load and cyclic external load.

2.3.1 Method 1

Method 1 is an LEFM approach and requires one linear FEA with two load steps. No crack face contact conditions were used in this method. The first Abaqus load step is with static load only (e.g. residual stress applied to the crack faces through the user subroutine DLOAD) . This provides a solution for point A in Figure 5. The second load step is with the maximum value of the cyclic external load only. This provides a solution for point D. The external load results can be scaled and combined with the static load result to give a solution at any desired external load level. This is the required Method 1 response line AB. This method can be used with constant amplitude or spectrum external loading. It is noted that Method 1 causes a reduction in R but no reduction in ΔK.

2.3.2 Method 2

Method 2 allows crack face contact to be incorporated in an analysis with combined static and cyclic loading. For crack growth prediction the method is currently limited to constant amplitude external loading in which results are generated at the minimum and maximum external loads in the cycle. For crack growth with spectrum loading it would be necessary to generate and use the complete non-linear curve of type CEB. However, it is possible to use Method 2 to generate the full curve CEB for each crack position.

In Method 2 the first Abaqus load step is with static load and minimum external load applied together. This minimum external load may be non-zero and provides a solution for the point of minimum external load in the load cycle. The second load step is with static load and maximum external load applied together and provides a solution for the point of maximum external load in the load cycle. It is noted that if the second load step is broken into a number of distinct points, the full curve (CEB) between minimum and maximum external load can be generated and the technique extended to analyse variable amplitude fatigue loading.

Without crack face contact this method produces a segment of the response line AB as generated from Method 1. If, however, crack face contact is included a non-linear “true solution” is obtained i.e. points between C and B. The stress intensity range for crack growth calculations can then be taken as the range between the two points.

It is important to note that the crack growth data in Figure 4 cannot be used directly in Method 2 with contact. Such experimental crack growth is usually constructed using standard LEFM 2-D stress intensity factors solutions for standard test specimens and ΔK calculated using the linear relationship $\Delta K = K_{max}(1-R)$ as in Method 1. Also, in the presence of contact, the response depends upon the external load level below point E in Figure 5.

These considerations are the subject of further research and all example fatigue crack growth analyses are restricted to the Method 1 LEFM approach.

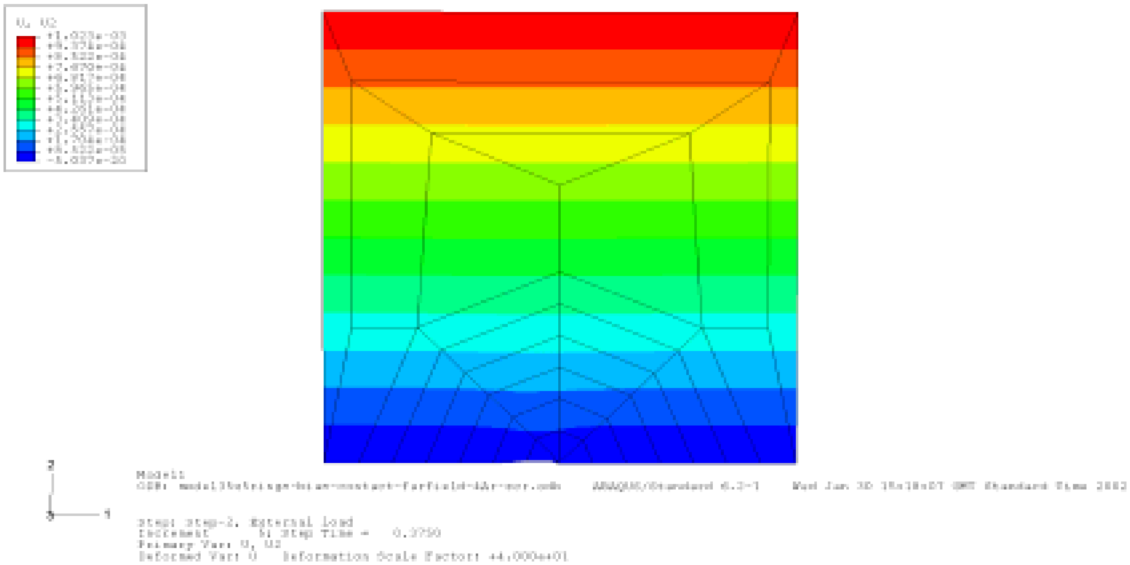
3 EXAMPLE 3-D FINITE ELEMENT ANALYSES

Three-dimension FEA have been conducted using the fracture mechanics Zencrack software (1) which is interfaced to Abaqus to analyses 3-D crack growth under generalized fatigue spectrum loading. With minimal user data, Zencrack can introduce crack front defects and advance the crack fronts under fatigue loading to user defined crack growth data. In these examples the loading was restricted to constant amplitude.

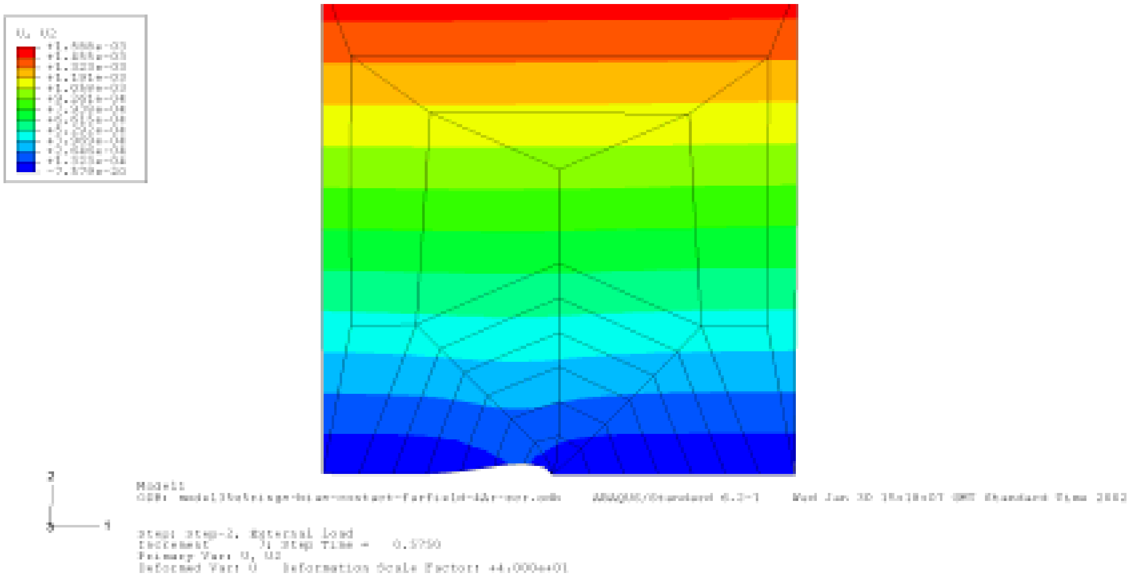
3.1 Single Edge notched specimen

The SEN specimen was used to generate the response curves typified in Figure 5 for a range of **static** external loads in the presence of residual stress with and without contact at the crack faces. The mesh density of the Zencrack “crack blocks” and was verified such that the DLOAD user subroutine generated a crack face pressure distribution that matched the specified residual stress distribution. From these results Methods 1 and 2 were developed and the mesh density of the Zencrack “crack blocks” were found to be adequate.

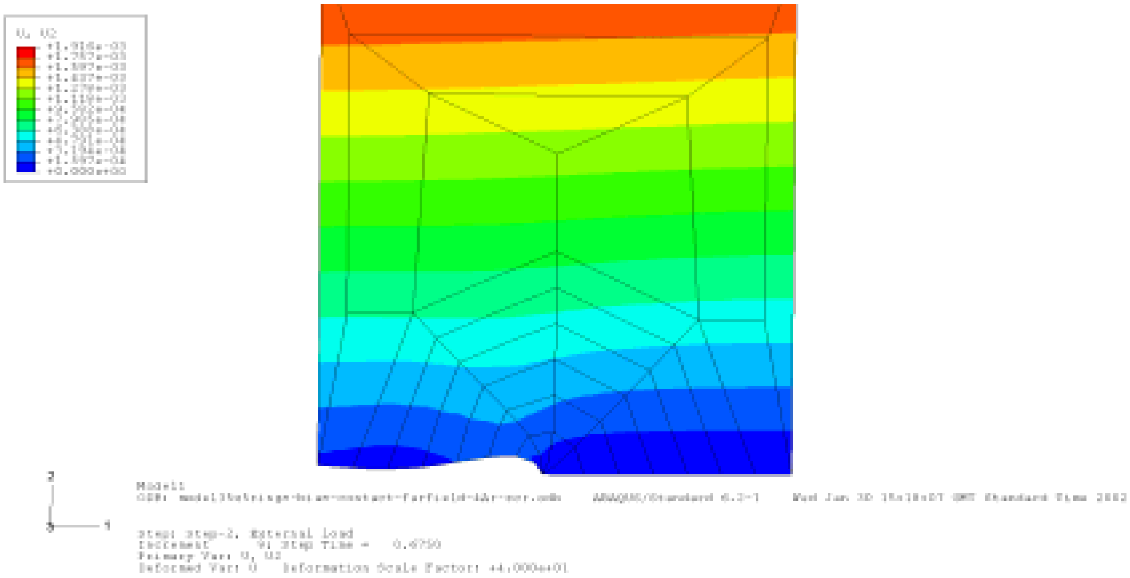
An important aspect of closure at the crack faces is the progressive opening of the crack faces leading to the curvilinear curve CE in Figure 5. The deformed shapes (with amplification x40) under increasing external loading of the SEN specimen are shown in the Figures 6 to 9.



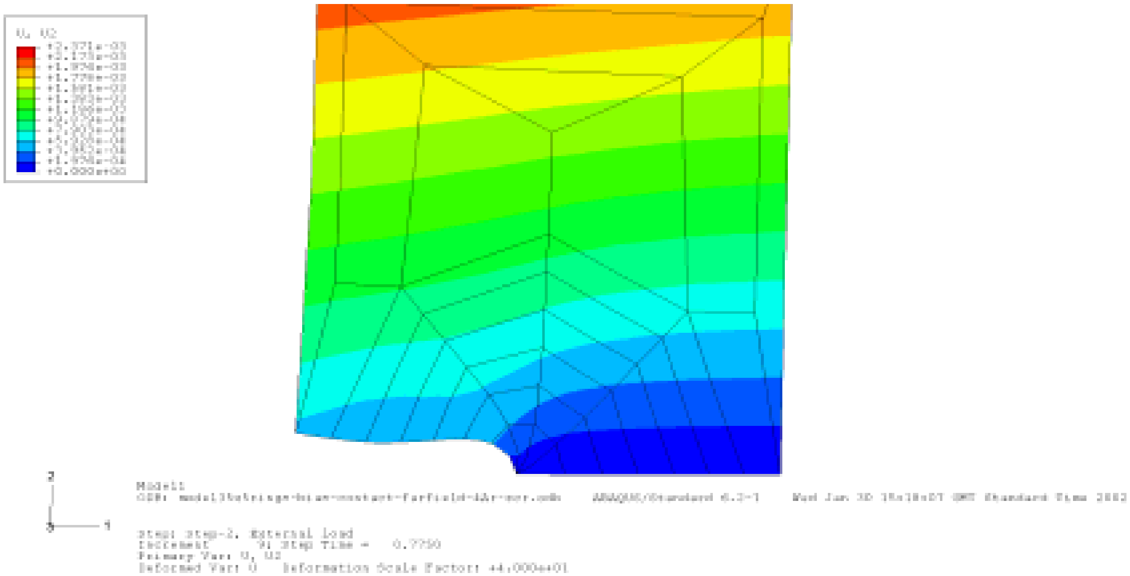
6. Figure 3-1 Crack face displacement at 562.5 MPa far-field stress



7. Figure 3-2 Crack face displacement at 862.5 MPa far-field stress



8. **Figure 3-3 Crack face displacement at 1012.5 MPa far-field stress**

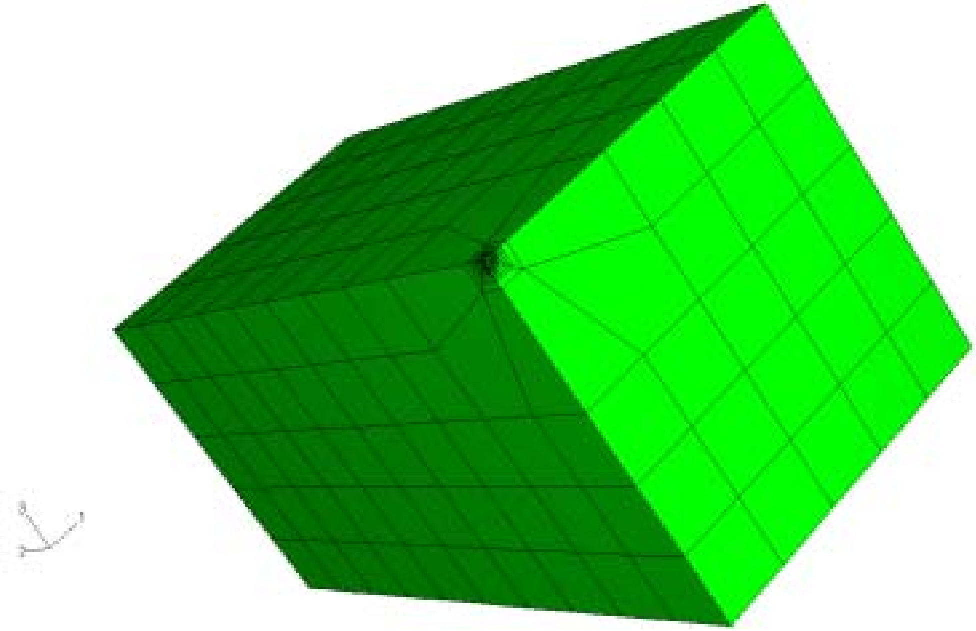


9. **Figure 3-4 Crack face displacement at 1162.5 MPa far-field stress**

3.2 Fatigue analysis of a semi-circular surface crack

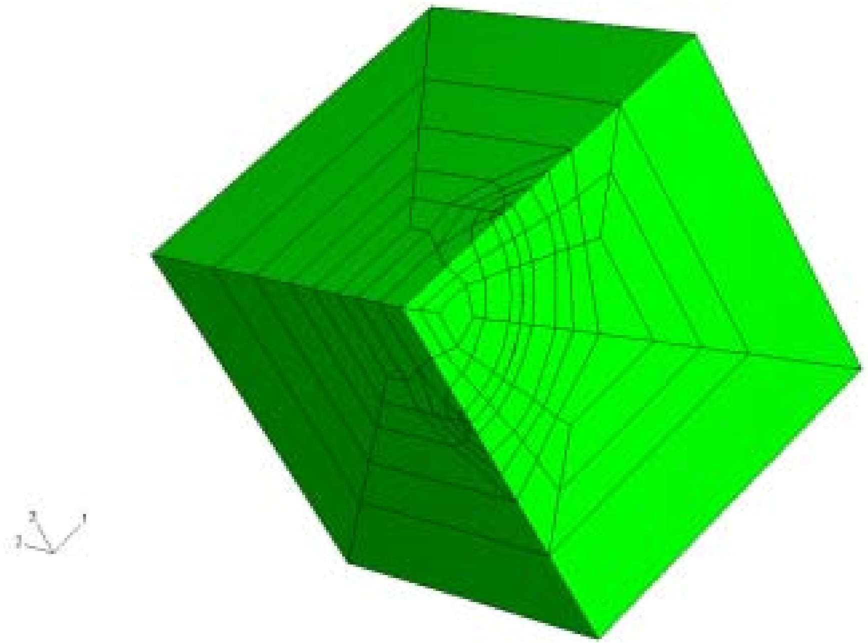
The finite element model (FEM) is a quarter-symmetry model of a tensile specimen with a semi-circular surface crack.

Dimensions: x-section thickness = 5 mm, width = 10 mm, half length = 20 mm
Initial crack size = 0.12 mm
Boundary conditions: constant amplitude 2-direction remote stress of 621 MPa (free to rotate)
Stress ratio = 0.1
¼ symmetry FEM



10. Initial geometry of a FEM including a semi-elliptic surface crack

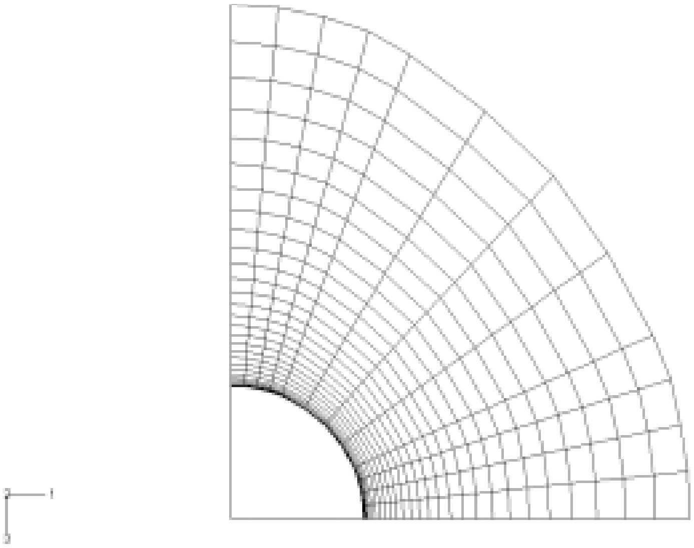
Zencrack has inserted a crack block into a regular mesh of the intact component. Notice the automatic shrinking of the crack block to maintain good aspect ratios in the crack block elements. Details of the crack block elements are shown in Figure 11.



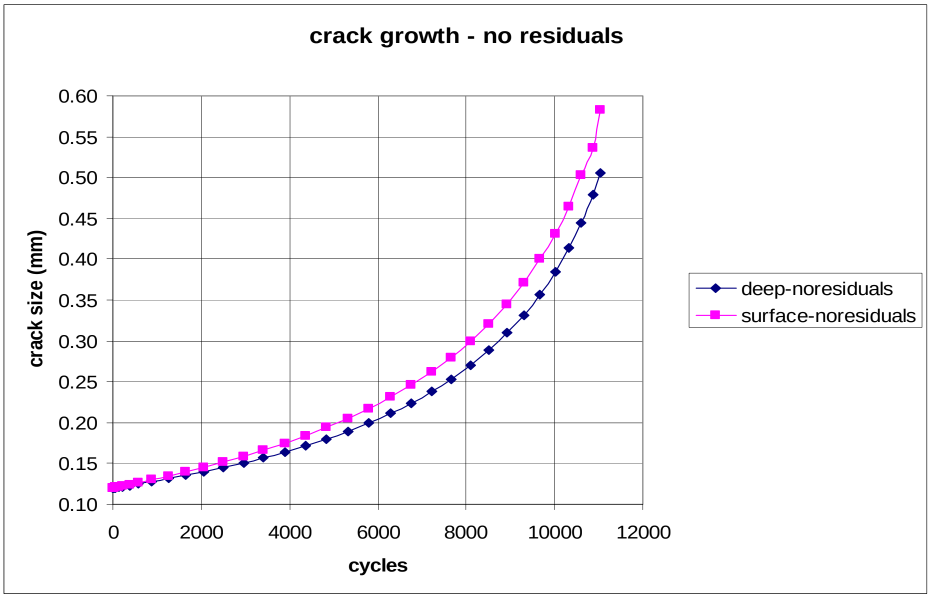
11. Details of the semi-elliptic FEM crack block

3.2.1 Crack growth without residual stresses

The crack growth profiles are shown in Figures 12 and 13 where 11035 cycles to failure were obtained. Please note that the depth is in 1-direction in the Figures.



12. Crack growth profile with no residual stresses



13. Crack growth with no residual stresses

3.2.2 Crack growth with residual stresses

The finite element analyses applies 50% of the residual stress distribution shown in Figure 1 to investigate the effect of stress relaxation during early service after component refurbishment using shot peening.

Two analyses were conducted using different tolerances (TOLA) of incremental crack growth between automatic re-meshing and finite element analyses. The tolerances were set at 0.002 mm and 0.005 mm to test convergence of the solutions. The results are shown in Figures 14 to 16.

Table 2 shows the number of cycles to failure are significantly greater than those obtained without residual stress and clearly demonstrate the benefits of shot peening surface treatment

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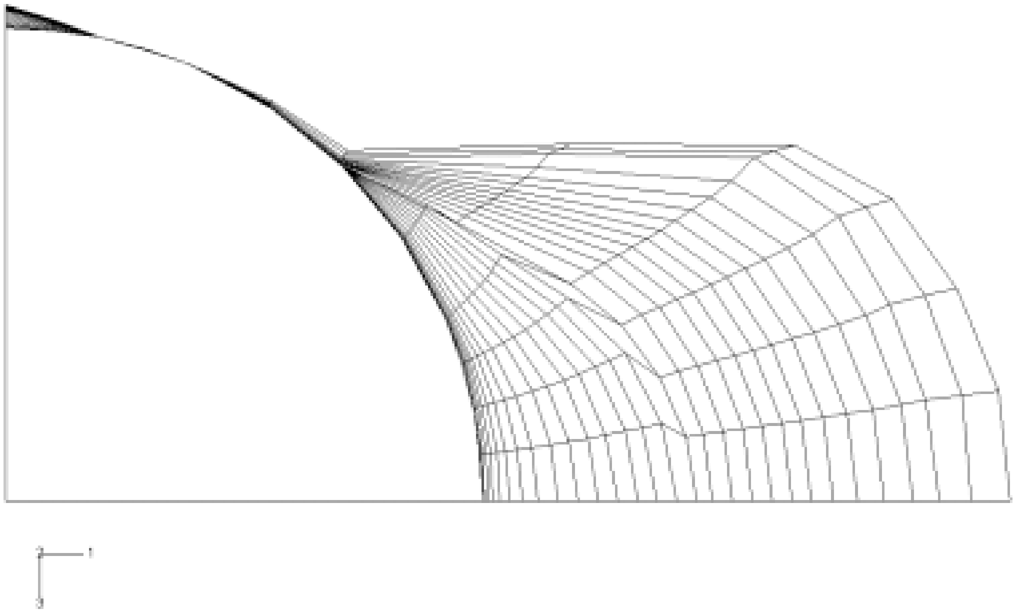
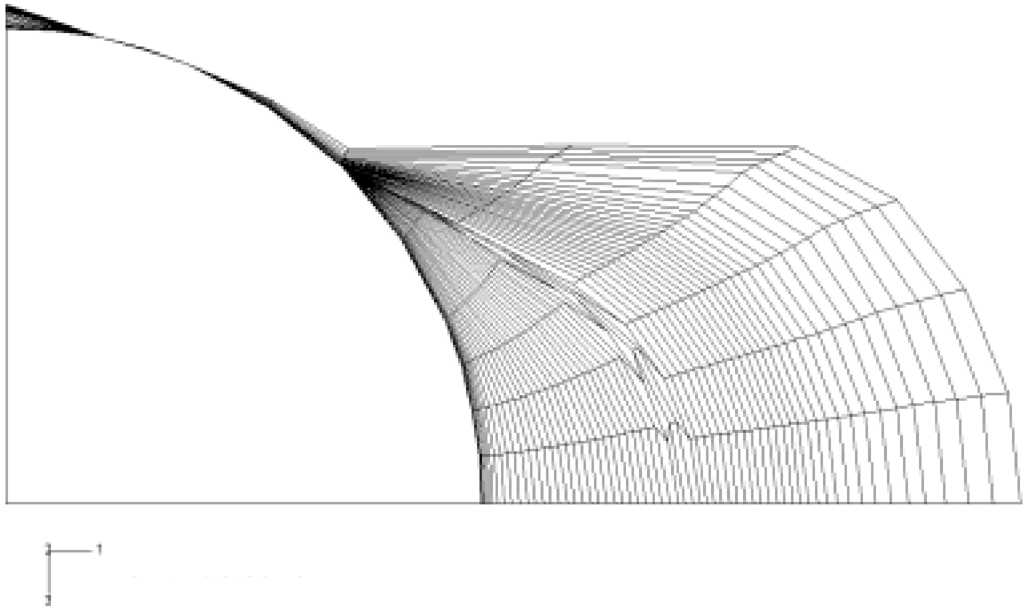
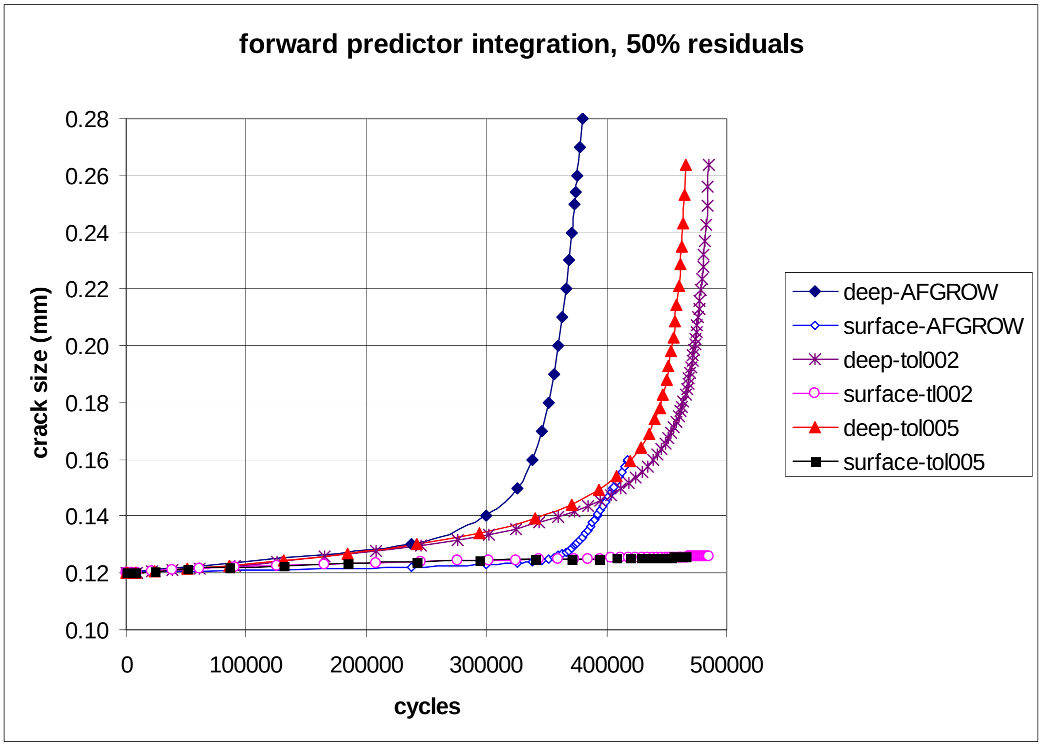


14. Crack growth profile – forward predictor, TOLA=0.005 mm



15. Crack growth profile – forward predictor, TOLA=0.002 mm



16. Zencrack LEFM analysis using forward predictor integration with residual stresses

Cycles to failure	Number of FEA	TOLA (mm)
485678	63	0.002
466231	32	0.005

2. Forward predictor life prediction with residual stresses

4 SUGGESTIONS FOR NEW ABAQUS CAPABILITIES FOR FUTURE DEVELOPMENT

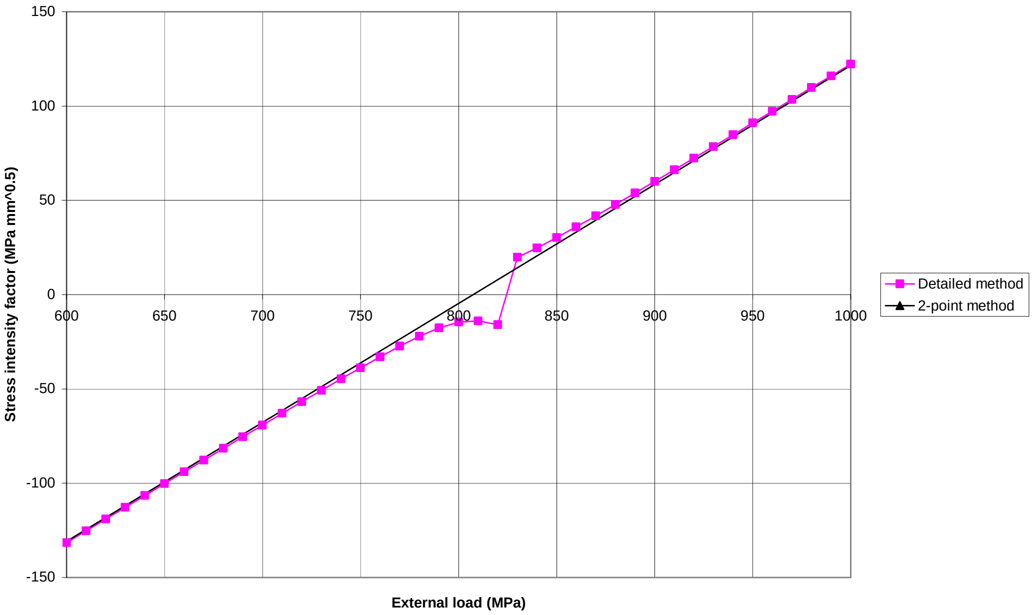
4.1 Oblique cracks and cracks which grow non-planar

The presented example analyses all deal with crack surfaces normal to the shot peened surface. If an initial oblique crack or crack growth becomes non-planar to the surface shear load components will be induced at the crack faces and will be resisted by friction forces on contacting areas of the crack surfaces.

It would be useful to pass the surface normal vectors at the integration points into the DLOAD user subroutine and to be able to specify surface shear tractions on element faces leading to mixed mode loading.

4.2 Accuracy of J-Integrals at near zero energy

It has been observed in developing the methods to superimpose residual stress and cyclic external load that Abaqus results are somewhat anomalous near zero strain energy. In the two point method the residual stress is applied in the first step and the “full” external load applied in a single step. In the detailed method the external cyclic loading is applied in increments (see Figure 17). This matter is being investigated by HKS.



17. Results for residual load magnitude varying with distance from crack front

5 CONCLUSIONS

The Abaqus DLOAD user subroutine provides sufficient functionality to model fatigue crack growth normal to the surface in the presence of residual stress distributions.

Such analyses are important in life extension programs to avoid early retirement of service components by surface treatments such as shot peening. Also inspection periods after refurbishment can be extended.

The presented fatigue growth analyses were limited to a conservative LEFM approach under constant amplitude loading which did not consider crack closure effects but provided a dramatic increase in service life.

Further research is being conducted to include crack closure to provide even more detailed enhancement to component fatigue life. This approach will require re-formulation of experimental fatigue crack growth data and introduction of the influence of external cyclic load magnitudes due to the non-linear response with crack closure. Current Zencrack capabilities can be used to conduct this re-formulation.

Suggestions have been proposed to extend the analysis capabilities in the DLOAD routine to analyse cracks which are not normal to the surface.

REFERENCES

1. Zentech International Ltd – Zencrack Version 7.0 User Manual and demonstration software download site, http://www.zentech.co.uk
2. Further background references can be found on the web site http://www.zentech.co.uk/tech_pap.htm